LEVEL II

FINITE ELEMENT ANALYSIS OF DYNAMIC CRACK PROPAGATION

Jaleez Ahmad

by

J. Jung, J. Ahmad and M. F. Kanninen

Battelle
Columbus Laboratories
Columbus, Ohio

and

C. H. Popelar
Engineering Mechanics Department
The Ohio State University
Columbus, Ohio

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ABSTRACT

A finite element analysis of stationary and propagating cracks in the presence of inertia forces is presented. An extension of the J-integral approach is employed. To model a propagating crack, a conceptually simple yet effective technique has been developed. The new crack propagation scheme eliminates the difficulties associated with the use of moving singular elements.

INTRODUCTION

It is generally accepted that a crack arrest methodology based on a dynamic view of crack propagation and arrest is more fundamental than the quasi-static approach to the problem [1]. Specifically, when inertia forces, stress-wave reflections, and rate-dependent fracture processes are dominant, quasi-static assumptions will generally underestimate the true crack driving force. In many applications, such as nuclear pressure vessel design and structures subjected to impact loading, dynamic effects can be important. In these cases analytical models based on the quasi-static assumption may lead to erroneous conclusions.

Inclusion of dynamic features in an analytical model undoubtedly leads to complications. In recent years the finite element method has emerged as an important tool that can be used to resolve at least some of the mathematical difficulties associated with the problem. Considerable progress has also been made toward understanding some of the basic concepts.

The purpose of the paper is to present the salient features of a recently developed finite element dynamic crack propagation modeling technique. Results of an elasto-dynamic analyses will be presented for both stationary and running cracks. Comparisons will be made with available experimental results. Through the analyses of test specimens, it will be demonstrated how this analytical model can be used to acquire a better understanding of the dynamic crack propagation phenomenon.

Dynamic Analysis and Numerical Integration

The familiar finite element discretized version of the equations of motion are simply written in the following form:

$$[M] \{\ddot{X}\} + [C] \{\dot{X}\} + [K] \{X\} = \{R\} \tag{1}$$

where

$[M]$ = mass matrix
$[C]$ = damping matrix
$[K]$ = stiffness matrix
$\{R\}$ = the external load vector
$\{\ddot{X}\}$, $\{\dot{X}\}$, $\{X\}$ = the displacement, velocity, and acceleration vectors respectively.

There are several methods that could be used to perform the direct numerical integrations of the equations. The method selected for this work was the Newmark implicit scheme [2]. For this method, the following approximations are used:

$$\dot{X}_{t+\Delta t} = \dot{X}_t + [(1-\delta)\ \ddot{X}_t + \delta \ddot{X}_{t+\Delta t}]\ \Delta t \tag{2}$$

$$X_{t+\Delta t} = X_t + \dot{X}_t\ \Delta t + [(1/2 - \alpha)\ \ddot{X}_t + \alpha\ \ddot{X}_{t+\Delta t}]\Delta t^2 \tag{3}$$

where α and δ are parameters that can be varied for accuracy and stability while Δt is the time step. From the above equations it is possible to write the equations of equilibrium for time, $t + \Delta t$, in terms of displacements, velocities, and accelerations at time, t.

$$\begin{aligned}[[K] + a_0[M] + a_1[C]]\ \{X_{t+\Delta t}\} = \{R_{t+\Delta t}\} &+ [M]\ (a_0\{X_t\} + a_2\{\dot{X}_t\} + a_3\{\ddot{X}_t\}) \\ &+ [C](a_1\{X_t\} + a_4\{\dot{X}_t\} + a_5\{\ddot{X}_t\})\end{aligned} \tag{4}$$

where

$$\begin{aligned}&a_0 = \frac{1}{\alpha\Delta t^2}\ ;\ a_1 = \frac{\delta}{\alpha\Delta t}\ ;\ a_2 = \frac{1}{\alpha\Delta t}\ ;\ a_3 = \frac{1}{2\alpha} - 1 \\ &a_4 = \frac{\delta}{\alpha} - 1\ ;\ a_5 = \frac{\Delta t}{2}\ (\frac{\delta}{\alpha} - 2)\ ;\ a_6 = \Delta t\ (1-\delta) \\ &a_7 = \delta\Delta t\ .\end{aligned} \tag{5}$$

Solving Equation (4) yields $\{X_{t+\Delta t}\}$ whereupon the accelerations and velocities at time, $t + \Delta t$, can be calculated using Equations (2) and (3).

Elasto-dynamic Analysis of Stationary Cracks

Consider the problem of determining the stress intensity factor for a stationary crack in a structure subjected to dynamic loadings. Several investigators have solved this problem by employing singular elements around the crack tip [3-5]. While this approach has proved to be successful, it will later require special considerations when the crack is propagating. An alternative to the singular element approach is to derive the stress intensity factors from a path independent J-integral [6]. After accounting for the inertia

effects, the rate of energy release per unit of crack advance in the direction of the crack X_1 is defined as:

$$J = \int_{\Gamma+\Gamma_s} [w dx_2 - \sigma_{ij} n_j \frac{\partial u_i}{\partial x_1} ds] + \iint_A \rho \ddot{u}_i (\frac{\partial u_i}{\partial x_1}) dA \quad , \qquad (6)$$

where the first integral is the conventional J-integral over an arbitrary path surrounding the crack tip and the second integral is an integral over the area, A, enclosed by the path, $\Gamma + \Gamma_s$; see Figure 1.

An application of this J-integral is shown in the following example in which a centrally cracked plate in plane strain is impulsively loaded by a uniform stress; see Figures 2 and 3. The finite element model employed 309 nodes and 90 eight noded quadratic isoparametric elements; see Figure 4. The material is linear elastic with a Young's modulus of 200 GPa, Poisson's ratio of 0.3 and density of 5 g/cm^3. Newmark's implicit time integration scheme was used ($\alpha = 0.25$ and $\delta = 0.5$) with a time step of 0.15 microseconds. A consistent mass formulation was used for this analysis.

The results of the present analysis are shown in Figure 5 along with those of other investigators for comparison [7,8]. As the figure shows, the present analysis is in excellent agreement with the other results. It should be mentioned that the path independence of this J-integral has been previously demonstrated [7].

An Analysis of Running Cracks

A technique has been developed that allows for the analysis of running cracks without the need for mesh adjustments or iterations. This technique is based on earlier crack propagation investigations using a finite difference based analysis [9]. It begins by subdividing the element immediately ahead of the crack tip into what can be thought of as subelements, as shown in Figure 6, (in this case, for a mesh composed of four noded linear elements). During propagation, the crack tip will be, in theory, allowed to move in discrete jumps along the crack plane through these subelements; e.g., the crack tip will move from Point "1" to Point "2" in one jump and later move

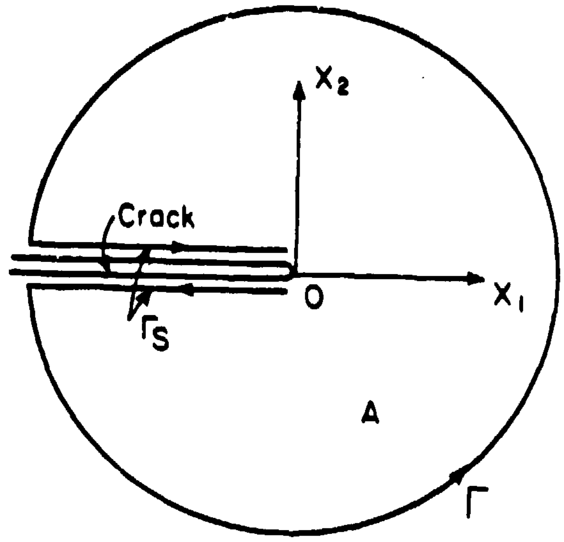

FIGURE 1. COORDINATE SYSTEM AND CURVES Γ and Γ_S.

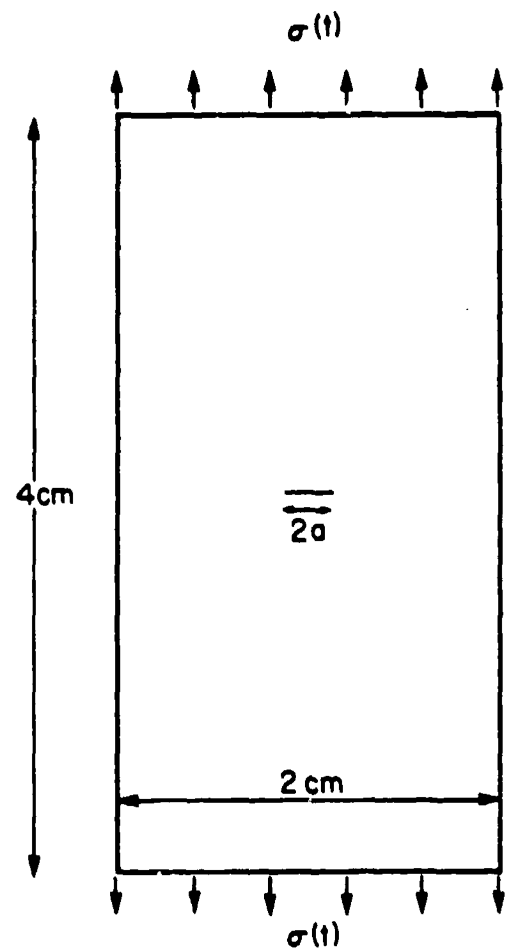

FIGURE 2. GEOMETRY FOR STATIONARY CRACK ANALYSIS; $a = 0.24$ cm.

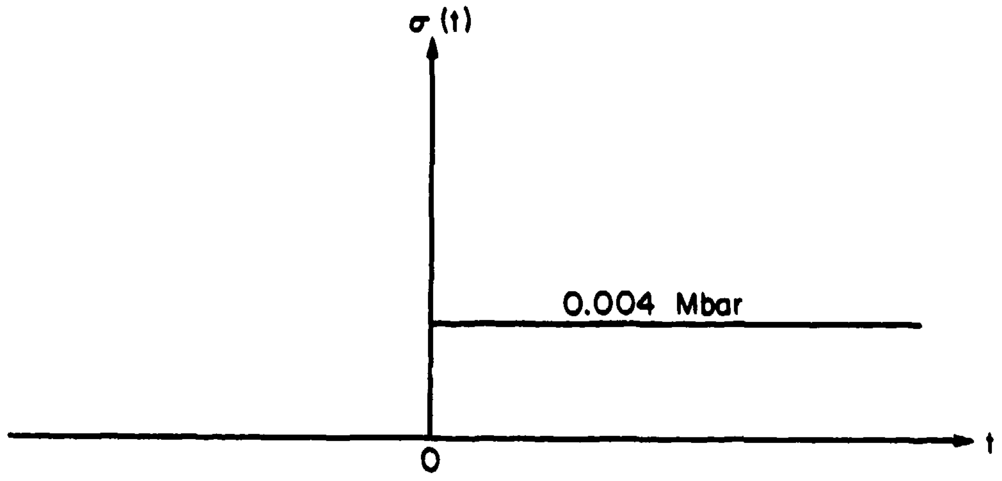

FIGURE 3. LOADING HISTORY FOR STATIONARY CRACK ANALYSIS.

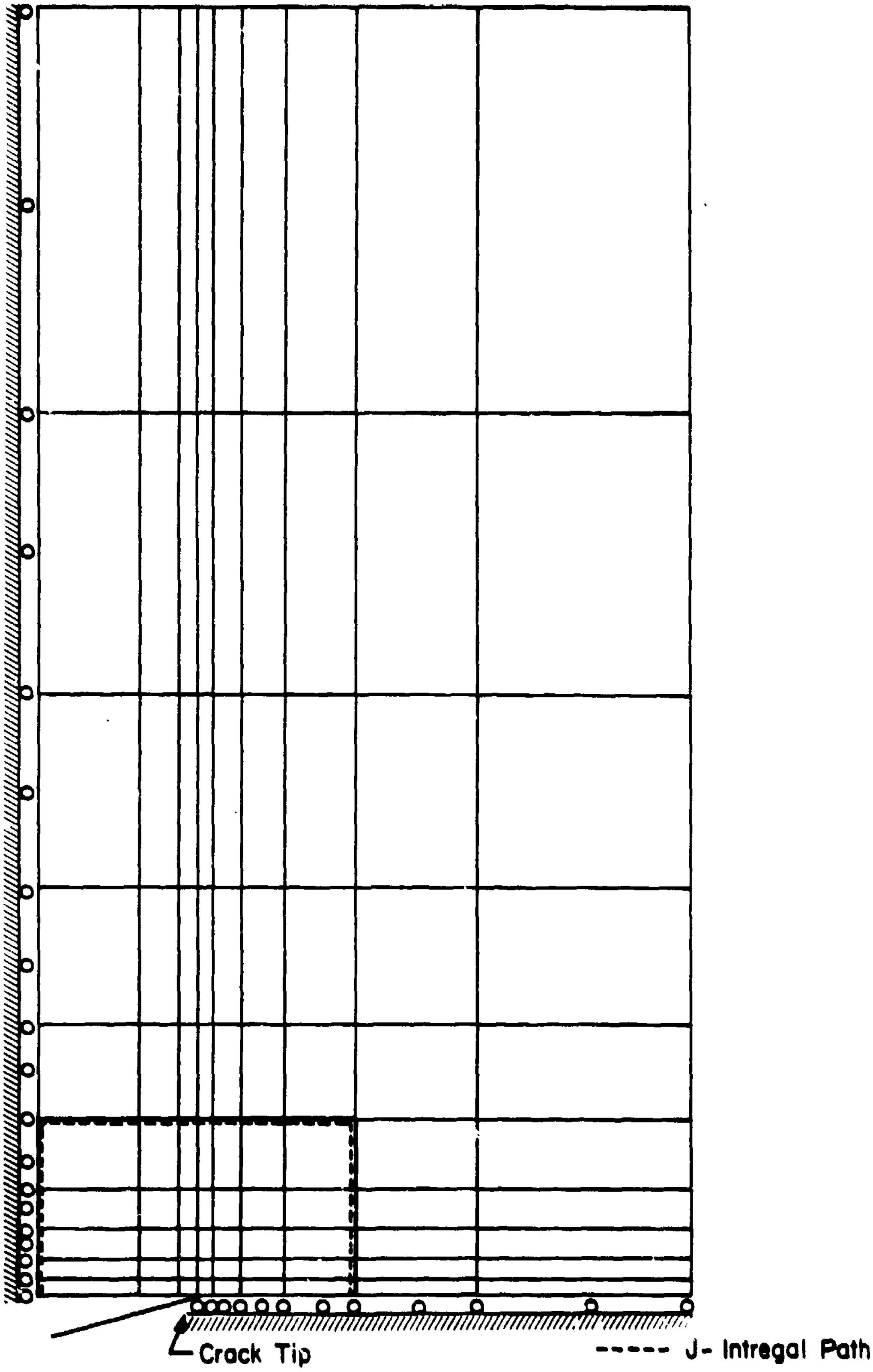

FIGURE 4. FINITE ELEMENT MESH AND J-INTEGRAL PATH FOR A STATIONARY CRACK PROBLEM

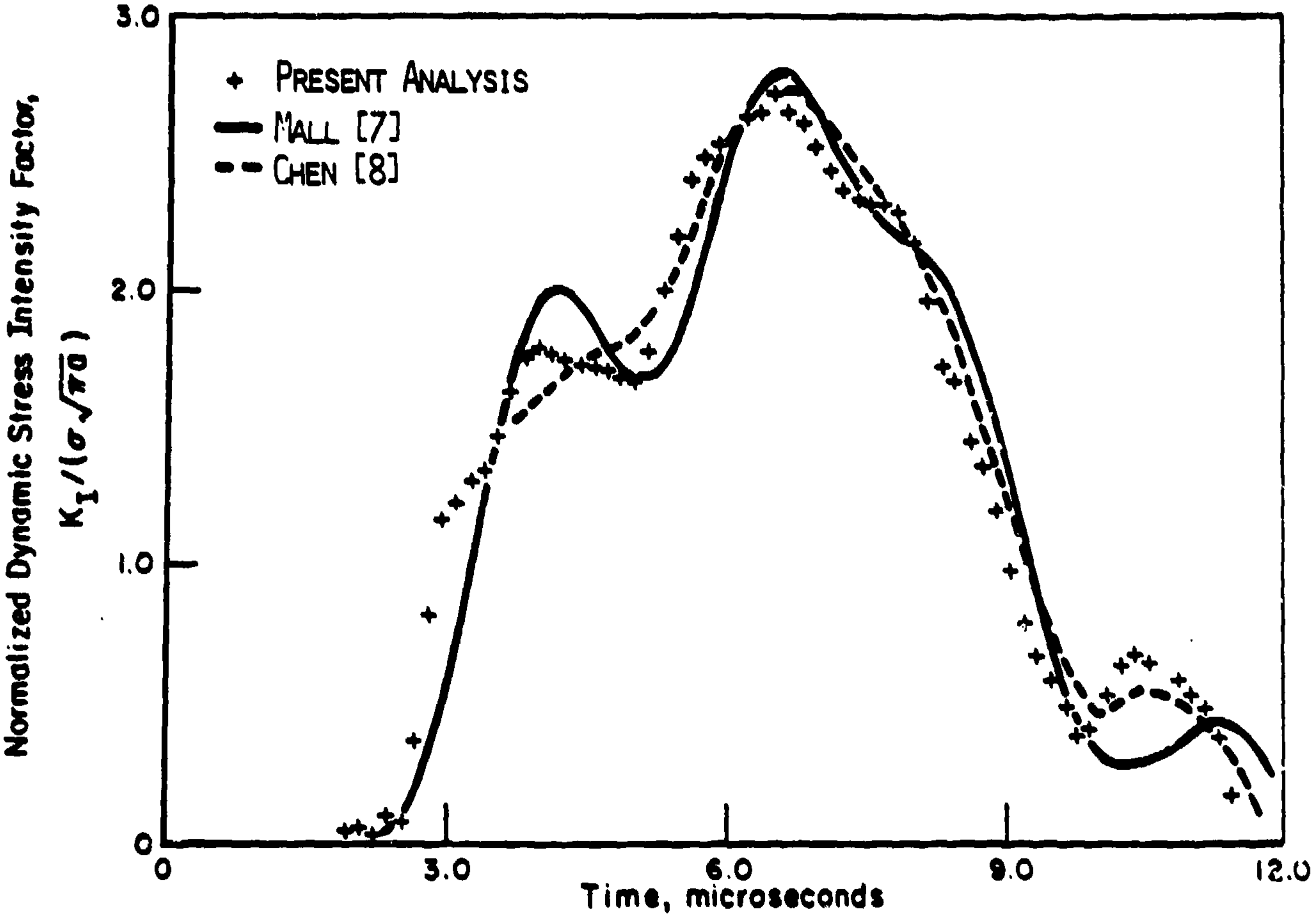

FIGURE 5. DYNAMIC STRESS INTENSITY FACTOR VERSUS TIME FOR AN IMPULSIVELY LOADED CENTER-CRACKED PANEL WITH A STATIONARY CRACK

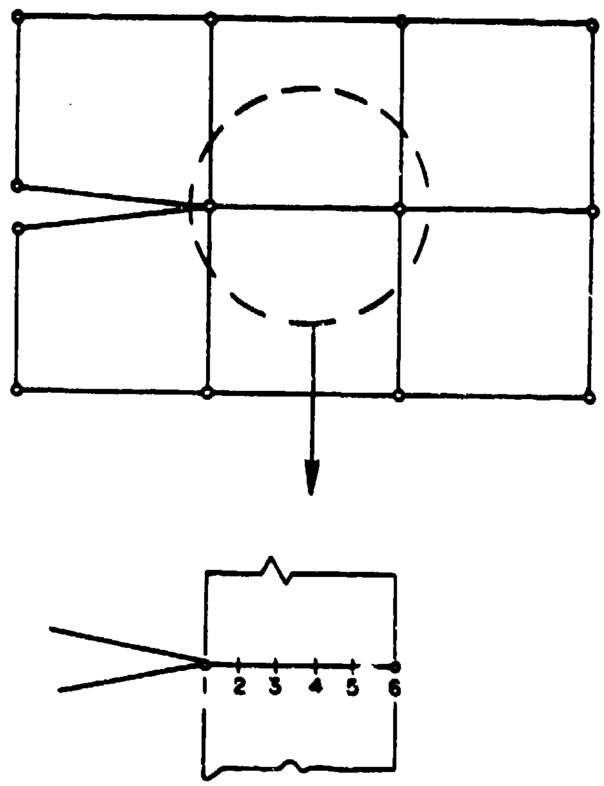

FIGURE 6. ELEMENT SUBDIVISION.

from Point "2" to Point "3" and so on, as the analysis dictates. The stress intensity factor at any crack location will be determined by a J-integral evaluation, Equation (6).

Consider the situation with the crack tip at Point "1". The crack velocity, V, will be estimated by counting the number of time steps, n, which are required until the stress intensity factor, K, is equal to the fracture toughness and performing the simple calculation:

$$V = \frac{|x(2) - x(1)|}{n\Delta t} \tag{7}$$

where Δt is the time step size. This algorithm is repeated as the crack tip moves from Point "1" to "2", "2" to "3", and so on.

This algorithm has the advantage of allowing the calculation of pseudo crack velocities after each time step. This pseudo velocity is equal to the actual crack velocity when the fracture criterion

$$K = K_D\ (V, T)$$

is satisfied, where the fracture toughness can be a function of the crack velocity, V, and material temperature, T. Since the crack velocity is calculated first and then the fracture toughness, there are no problems with using a velocity independent fracture toughness relation. This type of criterion would cause problems if the crack was propagating, i.e., $K = K_D$, and the inverse of the K_D relation was used to determine the crack velocity as is done in some codes.

It was then necessary to decide the number of subdivisions to make in each element. This was accomplished by determining the maximum crack velocity to be allowed in the analysis, V_{max}. In this study, V_{max} was taken as the bar wave speed, $\sqrt{E/\rho}$. Once V_{max} is set, the maximum distance the crack can propagate per time step, $\Delta x'$, is

$$\Delta x' = V_{max} \cdot \Delta t \quad . \tag{8}$$

The number of $\Delta x'$ units in an element length, Δx, is, $\frac{\Delta x}{\Delta x'}$. To make the number of subdivisions an integer value and to prevent the crack from traveling more than one element length per time step, the number of subdivisions, N, is taken to be

$$N = \frac{\Delta x}{\Delta x'} + 1 \tag{9}$$

where N is a truncated integer value. Should the crack want to propagate at one subdivision per time step, its propagation speed will be somewhat less than the maximum velocity, V_{max}, set previously due to the truncation. The amount by which the actual maximum crack velocity differs from V_{max} will depend on the value of N.

The last detail was to determine how the crack tip would be placed at the subdivision lines. This was conceptually performed by placing a force on the element to which the crack tip is adjacent. For a four-noded element, the force is placed on the one node on the crack plane behind the crack tip and for an eight-noded element nodal, forces would be placed on the two nodes as shown in Figure 7. The forces were postulated to be linearly related to the crack tip location by the following equation:

$$\frac{F_i}{F_{o_i}} = [1 - \frac{a}{\Delta x}] \tag{10}$$

where F_i is the force at node "i", F_{o_i} is the nodal force at node "i" just prior to the node release, as shown in Figure 7, "a" is the crack length in the element which the crack tip is in, and "Δx" is the length of that element. In the present study, eight-noded quadratic isoparametric elements were used. The midside node was released simultaneously with the trailing vertex node. The force history of each node followed the prescription given in Equation (10). Both nodes were released simultaneously to avoid possible problems discovered in another investigation [10].

A summary of the above algorithm for a quasi-statically initiated event is given in Figure 8.

In the algorithm just described, the location of the "crack tip" is obviously not actually known when there are forces on the crack face. The

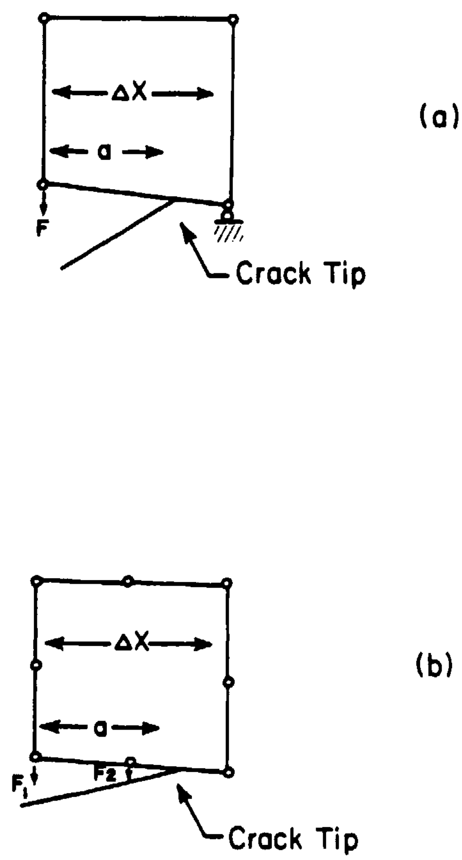

FIGURE 7. PLACEMENT OF NODAL FORCES FOR (a) A FOUR-NODED ELEMENT AND (b) AN EIGHT-NODED ELEMENT.

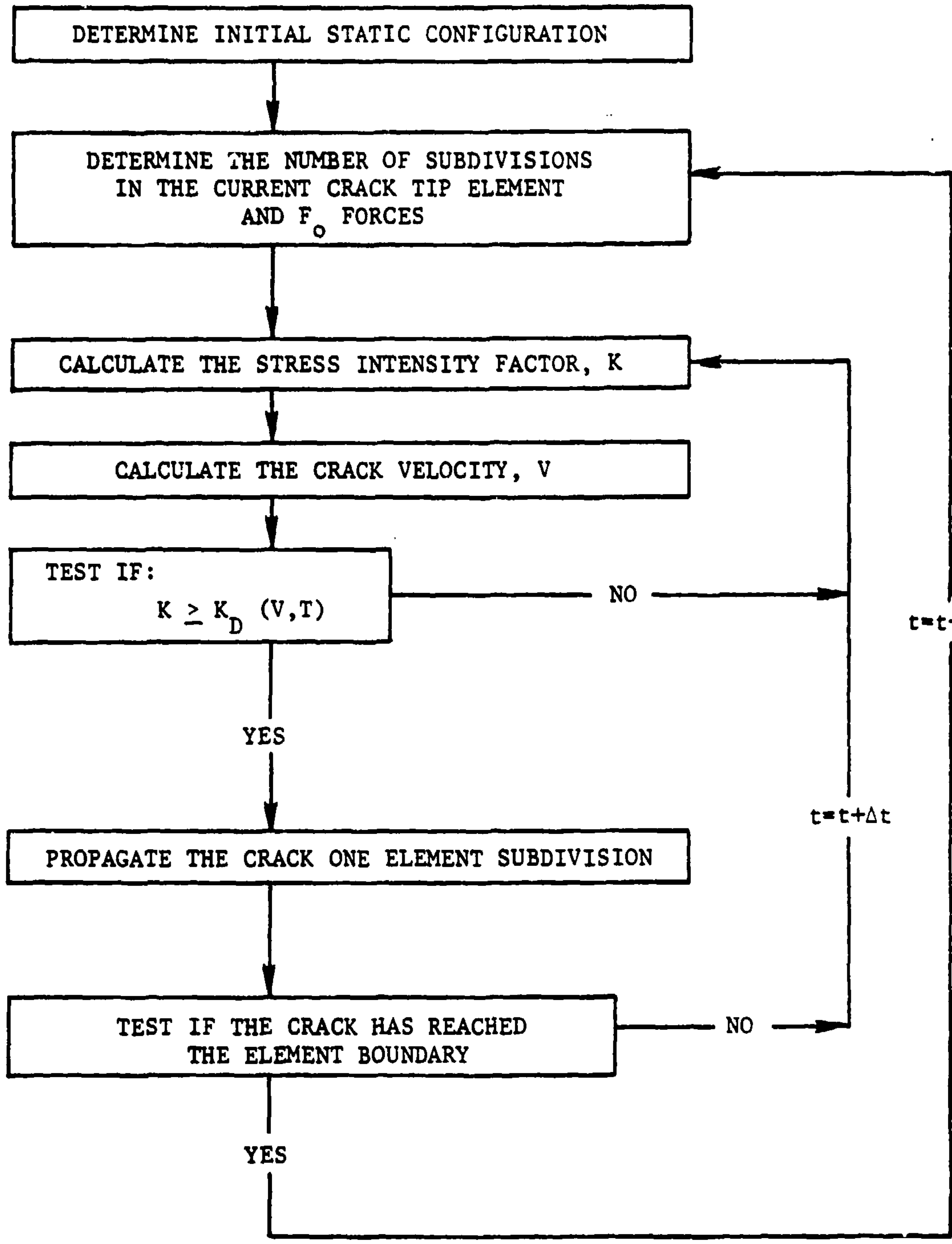

FIGURE 8. FLOW CHART OF PROPAGATION ALGORITHM FOR A QUASI-STATICALLY INITIATED DYNAMIC EVENT

only time when the location of the crack tip is unambiguous is when there are no forces on the crack face. But in order to avoid regeneration of the mesh (which would be necessary to place a node at the desired crack tip location) the interpretation used in the above algorithm was adopted. A similar interpretation has also been used by others [10,11,12].

The applicability of this interpretation can be somewhat tested by performing a constant velocity crack analysis. The problem chosen was a centrally cracked square plate of 40 mm x 40 mm with an initial crack length of 0.2 a/w, where w is the panel width. The panel was initially loaded with a uniform stress in the direction perpendicular to the crack. The properties of the plate were $E = 7716$ kgf/mm^2; $\nu = 0.286$; and $\rho = 2.5 \times 10^{-10}$ kgf $\cdot$ s^2/mm^4. The crack was propagated at a velocity of 0.2 of the shear wave speed of the material which was $C_s = 3.461 \times 10^6$ mm/sec. This problem is similar to that addressed by Broberg [13], except that Broberg treated the crack as opening from a zero initial length. The mesh used for the problem is shown in Figure 9. The model employed 213 nodes and 60 eight-noded isoparametric elements. The time integration employed a time step of 0.2887 microsecond with $\alpha = 0.25$ and $\delta = 0.5$ The results of the analysis is shown in Figure 10 along with the Broberg solution and solutions of Atluri [14]. The figure shows that the two computed solutions are very similar and they converge to the Broberg solution after the initial transient conditions have past. Hence, very reasonable estimates of the stress intensity factor with time can be obtained for the problem of a crack with prescribed velocity.

The inverse of the problem performed above is perhaps of even more importance; i.e., a problem in which the dynamic fracture toughness is specified and the crack length time history is sought. The ability of the proposed procedure to perform such an analysis was tested by comparing computed results with experimental data for a quasi-statically initiated crack in a 4340 steel three-point bend (dynamic tear) specimen; see Figure 11. The finite element mesh used for the analysis used 314 nodes and 91 eight-noded isoparametric elements; see Figure 12. A previously determined dynamic fracture toughness relation given by $K_{ID} = 65 + 0.44$ V was used [15].

The results of the analysis given in Figure 13 show excellent agreement with experimental results.

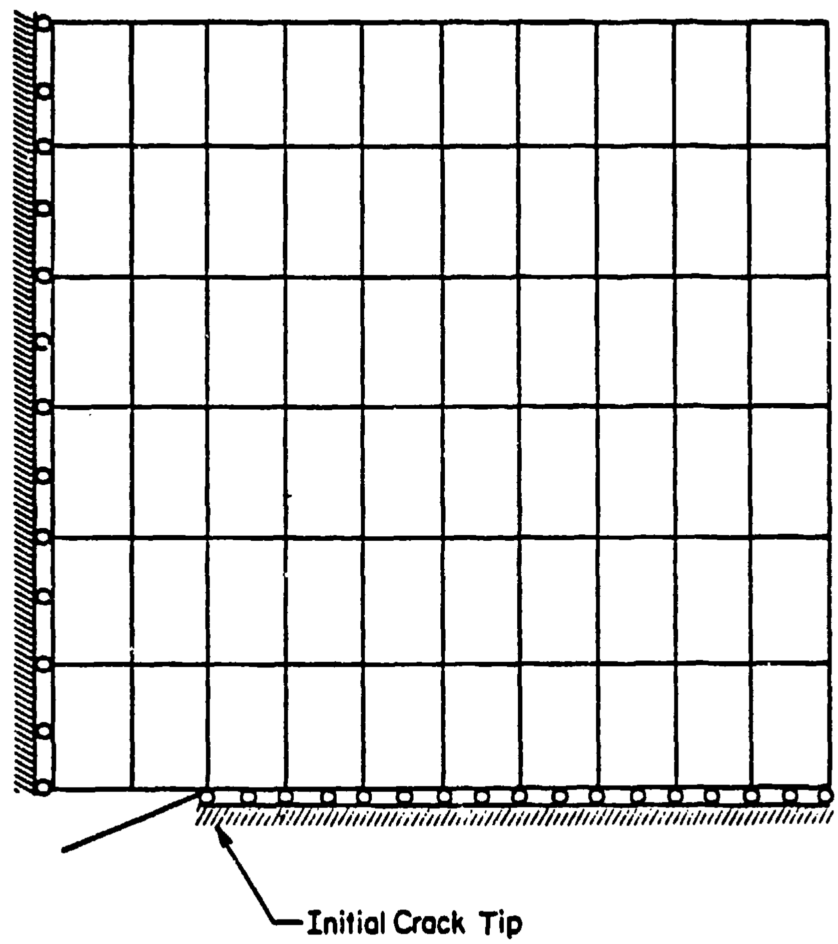

FIGURE 9. MESH FOR A CONSTANT VELOCITY CRACK PROBLEM

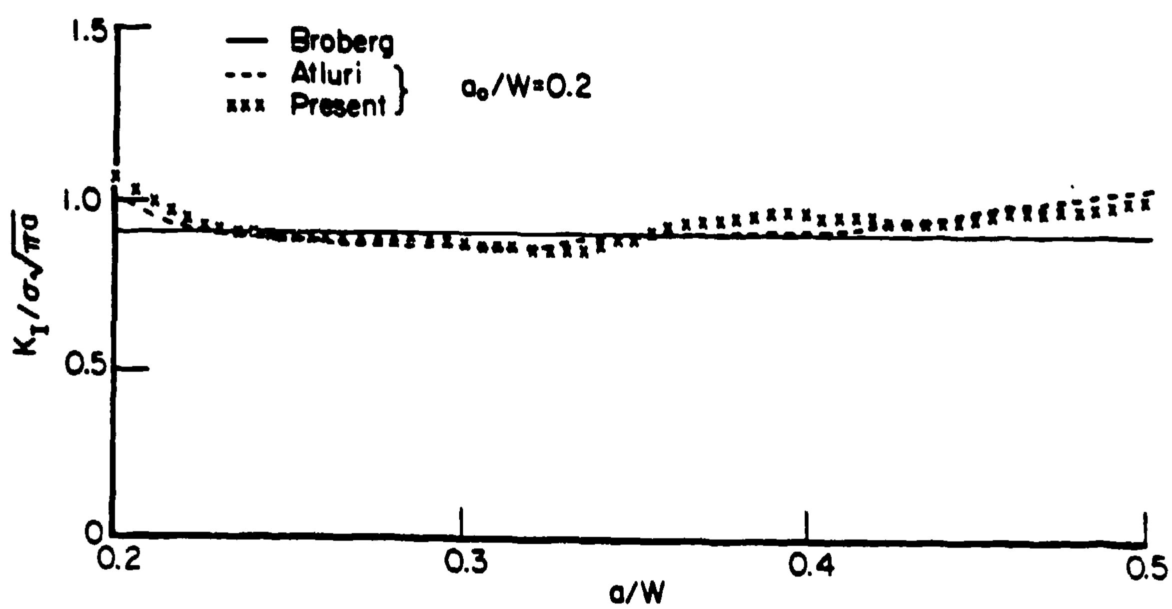

FIGURE 10. STRESS INTENSITY FACTOR FOR A CRACK STARTING AT $a_o/W = 0.2$ AND PROPAGATING AT A CONSTANT VELOCITY OF $V/C_S = 0.2$.

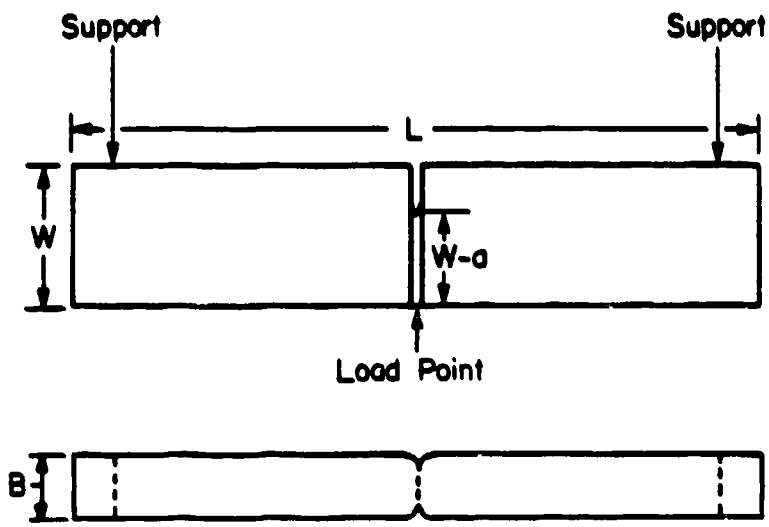

FIGURE 11. SPECIMEN GEOMETRY FOR A QUASI-STATICALLY INITIATED CRACK PROPAGATION TEST L = 181 mm, W = 38 mm, B = 15.8 mm, W-a = 28.5 mm.

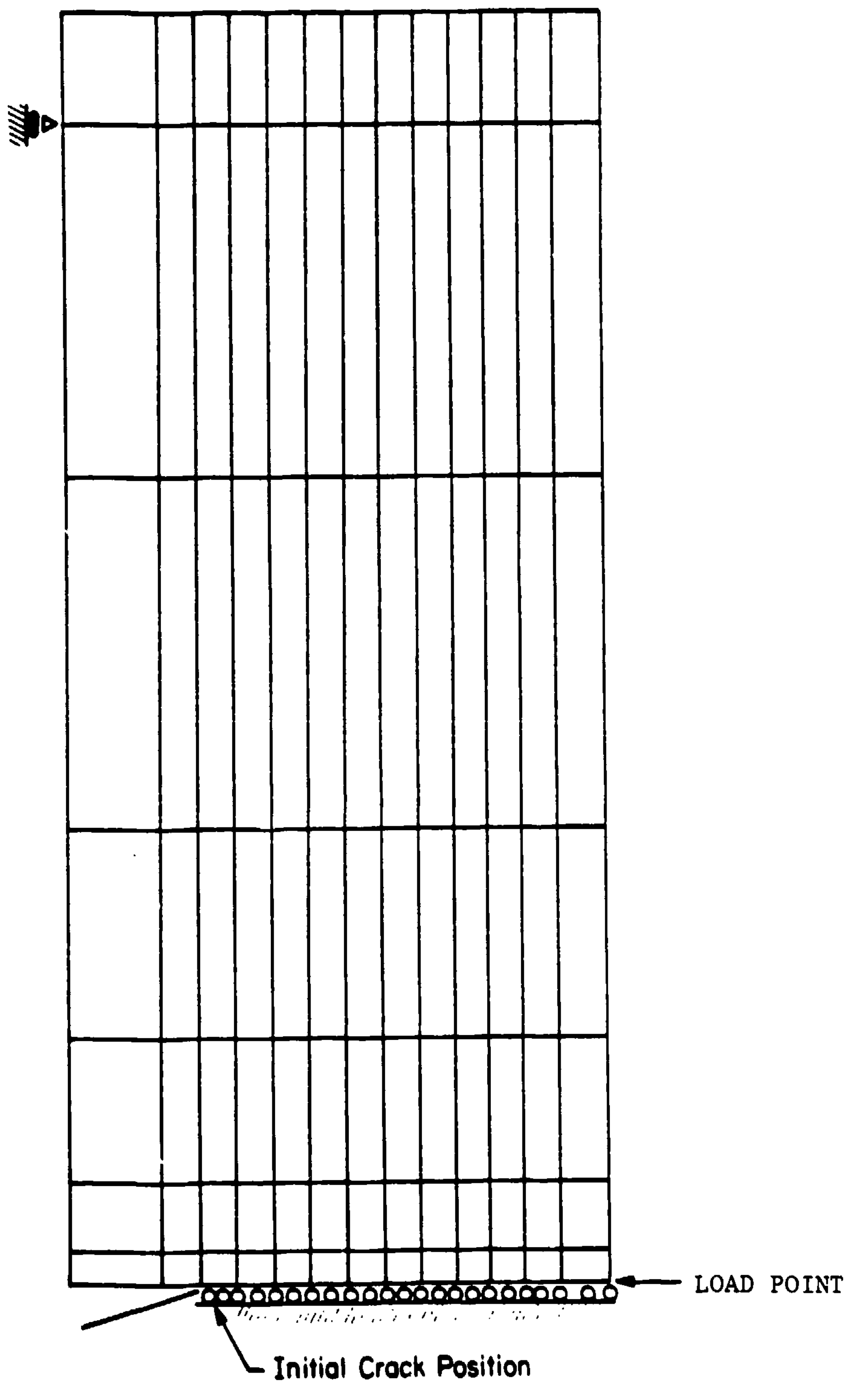

FIGURE 12. MESH FOR IMPACT LOADED THREE-POINT BEND ANALYSIS.

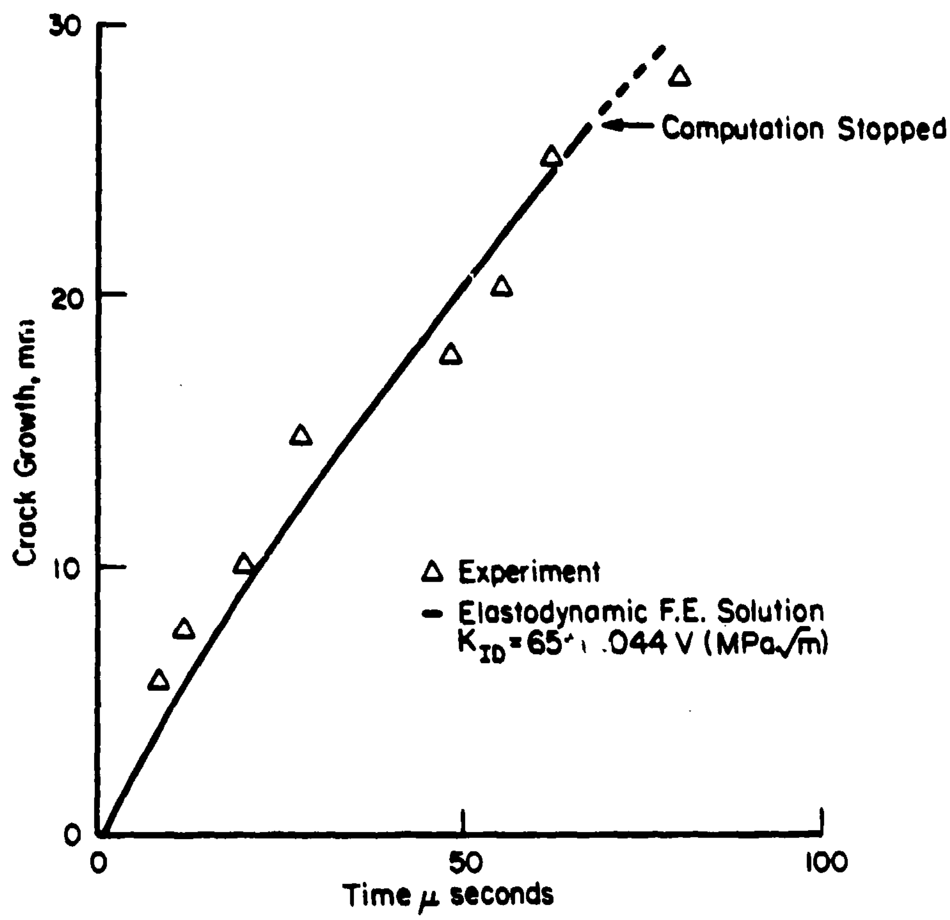

FIGURE 13. A COMPARISON OF ANALYSIS AND EXPERIMENT FOR A QUASI-STATICALLY INITIATED CRACK IN A 4340 DYNAMIC TEAR SPECIMEN.

CONCLUSIONS

A simple yet effective technique has been developed to perform dynamic crack propagation problems. The technique does not require using singular elements or updating the finite element mesh during crack propagation. Also, the dynamic fracture criterion need not be velocity dependent. Good agreement has been obtained between analytical results and experimental data.

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